第 3 章§ 1 2 重積分

p.72 練習問題 1-A

1. (1) 与式 $=\displaystyle\int_0^1\left\{\int_{2x^2}^{2x}(3y^2-xy)dy\right\}dx=\int_0^1\left[y^3-\frac{xy^2}{2}\right]_{2x^2}^{2x}dx=\int_0^1\{(8x^3-2x^3)-(8x^6-2x^5)\}dx$

$=\displaystyle\int_0^1(-8x^6+2x^5+6x^3)dx=\left[-\frac{8}{7}x^7+\frac{2}{6}x^6+\frac{6}{4}x^4\right]_0^1=-\frac{8}{7}+\frac{1}{3}+\frac{3}{2}=\underline{\frac{29}{42}}$

(2) 与式 $=\displaystyle\int_1^2\left\{\int_0^1\frac{x}{(x+y)^2}dy\right\}dx=\int_1^2\left\{\int_0^1\{x(x+y)^{-2}dy\right\}dx=\int_1^2x\left[-(x+y)^{-1}\right]_0^1dx=\int_1^2x\{-(x+1)^{-1}+x^{-1}\}dx$

$=\displaystyle\int_1^2\left(-\frac{x}{x+1}+1\right)dx=\int_1^2\frac{1}{x+1}dx=[\log(x+1)]_1^2=\log 3-\log 2=\underline{\log\frac{3}{2}}$

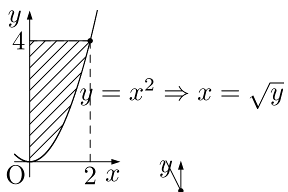
(3) 与式 $=\displaystyle\int_0^{\frac{\pi}{2}}\left\{\int_x^{2x}\sin(2x+y)dy\right\}dx=\int_0^{\frac{\pi}{2}}[-\cos(2x+y)]_x^{2x}dx=\int_0^{\frac{\pi}{2}}(-\cos 4x+\cos 3x)dx$

$=\displaystyle\left[-\frac{\sin 4x}{4}+\frac{\sin 3x}{3}\right]_0^{\frac{\pi}{2}}=-\frac{\sin 2\pi-\sin 0}{4}+\frac{\sin\frac{3}{2}\pi-\sin 0}{3}=\underline{-\frac{1}{3}}$

(4) 与式 $=\displaystyle\int_0^1\left\{\int_y^{\sqrt{y}}x^2dx\right\}dy=\int_0^1\left[\frac{x^3}{3}\right]_y^{\sqrt{y}}dx=\int_0^1\frac{y\sqrt{y}-y^3}{3}dy=\frac{1}{3}\left[\frac{2y^{\frac{5}{2}}}{5}-\frac{y^4}{4}\right]_0^1=\frac{1}{3}\left(\frac{2}{5}-\frac{1}{4}\right)=\underline{\frac{1}{20}}$

2. $D:0\leqq y\leqq 2,\ \dfrac{y}{2}\leqq x\leqq 3-y$ よって$\displaystyle\iint_D y^2dxdy=\int_0^2\left\{\int_{\frac{y}{2}}^{3-y}y^2dx\right\}dy=\int_0^2[y^2x]_{\frac{y}{2}}^{3-y}dy=\int_0^2y^2\left(3-y-\frac{y}{2}\right)dy$

$=\displaystyle\int_0^2\left(3y^2-\frac{3y^3}{2}\right)dy=\left[y^3-\frac{3y^4}{8}\right]_0^2=8-6=\underline{2}$

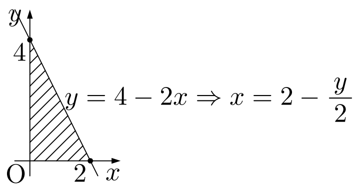


3. (1) $D:0\leqq x\leqq 2,\ x^2\leqq y\leqq 4\Rightarrow 0\leqq y\leqq 4,\ 0\leqq x\leqq\sqrt{y}$

$\displaystyle\int_0^2\left\{\int_{x^2}^4f(x,y)dy\right\}dx=\int_0^4\left\{\int_0^{\sqrt{y}}f(x,y)dx\right\}dy$

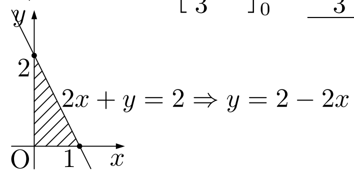
(2) $D:0\leqq y\leqq 4,\ 0\leqq x\leqq 2-\dfrac{y}{2}\Rightarrow 0\leqq x\leqq 2,\ 0\leqq y\leqq 4-2x$

$\displaystyle\int_0^4\left\{\int_0^{2-\frac{y}{2}}f(x,y)dx\right\}dy=\int_0^2\left\{\int_0^{4-2x}f(x,y)dy\right\}dx$



4. 曲面 $z=\sqrt{xy}$ と平面 $z=0$ の交線は $\sqrt{xy}=0$ より $x=0,y=0$ よって $D:0\leqq x\leqq 2,\ 0\leqq y\leqq 3$

$V=\displaystyle\iint_D\sqrt{xy}dxdy=\int_0^2\left\{\int_0^3\sqrt{xy}dy\right\}dx=\int_0^2\sqrt{x}\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^3dx=\int_0^2\frac{2}{3}\cdot 3\sqrt{3}\sqrt{x}dx=2\sqrt{3}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2=\underline{\frac{8\sqrt{6}}{3}}.$



5. $D:0\leqq x\leqq 1,\ 0\leqq y\leqq 2-2x(=2(1-x))$

$V=\displaystyle\iint_D(x^2+xy+y^2)dxdy=\int_0^1\left\{\int_0^{2(1-x)}(x^2+xy+y^2)dy\right\}dx$

$=\displaystyle\int_0^1\left[x^2y+\frac{xy^2}{2}+\frac{y^3}{3}\right]_0^{2(1-x)}dx=\int_0^1\left\{2x^2(1-x)+2x(1-x)^2+\frac{8(1-x)^3}{3}\right\}dx$

$=\displaystyle\int_0^1\left\{2x(1-x)\{x+(1-x)\}+\frac{8(1-x)^3}{3}\right\}dx=\int_0^1\left\{2x-2x^2+\frac{8(1-x)^3}{3}\right\}dx.$

ここで $\dfrac{8(1-x)^3}{3}$ について $1-x=t$ とおくと $-dx=dt$ より $dx=-dt$

よって$\displaystyle\int\frac{8(1-x)^3}{3}dx=\int\frac{8t^3}{3}(-dt)=-\frac{2t^4}{3}=-\frac{2(1-x)^4}{3}$ だから

$V=\displaystyle\left[x^2-\frac{2x^3}{3}-\frac{2(1-x)^4}{3}\right]_0^1=\left(1-\frac{2}{3}-0\right)-\left(0-0-\frac{2}{3}\right)=\underline{1}$