ドリル基礎数学

p. 166 83 三角関数の加法定理

83.1 (1) $\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\sin\frac{\pi}{4}\cos\frac{\pi}{6}-\cos\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$

(2) $\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}+\sqrt{2}}{4}$

(3) $\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\frac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}}=\frac{1-\frac{1}{\sqrt{3}}}{1+1\cdot\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}+1)^2}{3-1}=2-\sqrt{3}$

83.2 $\sin^2\alpha=1-\cos^2\alpha=1-\left(-\frac{4}{5}\right)^2=\frac{9}{25}$ α 第 3 象限の角より $\sin\alpha<0.\therefore\sin\alpha=-\frac{3}{5}$

$\cos^2\beta=1-\sin^2\beta=1-\left(\frac{5}{13}\right)^2=\frac{144}{169}$ β 第 2 象限の角より $\cos\beta<0.\therefore\cos\beta=-\frac{12}{13}$

(1) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta=\left(-\frac{3}{5}\right)\cdot\left(-\frac{12}{13}\right)+\left(-\frac{4}{5}\right)\cdot\left(\frac{5}{13}\right)=\frac{16}{65}$

(2) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\left(-\frac{4}{5}\right)\cdot\left(-\frac{12}{13}\right)-\left(-\frac{3}{5}\right)\cdot\left(\frac{5}{13}\right)=\frac{63}{65}$

(3) $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{-\frac{3}{5}}{-\frac{4}{5}}=\frac{3}{4},\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{\frac{5}{13}}{-\frac{12}{13}}=-\frac{5}{12}$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{3}{4}+\left(-\frac{5}{12}\right)}{1-\frac{3}{4}\cdot\left(-\frac{5}{12}\right)}=\frac{16}{63}$$

83.3 $\sin^2\alpha=1-\cos^2\alpha=1-\left(\frac{1}{3}\right)^2=\frac{8}{9}$ α 第 4 象限の角より $\sin\alpha<0.\therefore\sin\alpha=-\frac{2\sqrt{2}}{3}$

$\cos^2\beta=1-\sin^2\beta=1-\left(-\frac{1}{4}\right)^2=\frac{15}{16}$ β 第 3 象限の角より $\cos\beta<0.\therefore\cos\beta=-\frac{\sqrt{15}}{4}$

(1) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta=\left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(-\frac{\sqrt{15}}{4}\right)-\frac{1}{3}\cdot\left(-\frac{1}{4}\right)=\frac{2\sqrt{30}+1}{12}$

(2) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{1}{3}\cdot\left(-\frac{\sqrt{15}}{4}\right)+\left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(-\frac{1}{4}\right)=\frac{2\sqrt{2}-\sqrt{15}}{12}$

(3) $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{-\frac{2\sqrt{2}}{3}}{\frac{1}{3}}=-2\sqrt{2},\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{-\frac{1}{4}}{-\frac{\sqrt{15}}{4}}=\frac{1}{\sqrt{15}}$

$$\tan(\alpha-\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{-2\sqrt{2}-\frac{1}{\sqrt{15}}}{1+(-2\sqrt{2})\cdot\frac{1}{\sqrt{15}}}=-\frac{2\sqrt{30}+1}{\sqrt{15}-2\sqrt{2}}=-\frac{(2\sqrt{30}+1)(\sqrt{15}+2\sqrt{2})}{(\sqrt{15}-2\sqrt{2})(\sqrt{15}+2\sqrt{2})}$$

$$=-\frac{30\sqrt{2}+\sqrt{15}+8\sqrt{15}+2\sqrt{2}}{15-8}=-\frac{9\sqrt{15}+32\sqrt{2}}{7}$$

84.1 $\sin^2\alpha=1-\cos^2\alpha=1-\left(-\frac{3}{5}\right)^2=\frac{16}{25}$ α 第 3 象限の角より $\sin\alpha<0.\therefore\sin\alpha=-\frac{4}{5}$

(1) $\sin2\alpha=2\sin\alpha\cos\alpha=2\cdot\left(-\frac{4}{5}\right)\cdot\left(-\frac{3}{5}\right)=\frac{24}{25}$

(2) $\cos2\alpha=\cos^2\alpha-\sin^2\alpha=\left(-\frac{3}{5}\right)^2-\left(-\frac{4}{5}\right)^2=-\frac{7}{25}$

(3) $\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}=\frac{1-(-\frac{3}{5})}{2}=\frac{4}{5}$. α 第 3 象限より $\pi<\alpha<\frac{3}{2}\pi$ よって $\frac{\pi}{2}<\frac{\alpha}{2}<\frac{3}{4}\pi$. よって

$\sin\frac{\alpha}{2}>0$ で $\sin\frac{\alpha}{2}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$

(4) $\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}=\frac{1+(-\frac{3}{5})}{2}=\frac{1}{5}$. $\frac{\pi}{2}<\frac{\alpha}{2}<\frac{3}{4}\pi$ より $\cos\frac{\alpha}{2}<0$ で $\cos\frac{\alpha}{2}=-\frac{1}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$

84.2 $\tan^2\alpha+1=\frac{1}{\cos^2\alpha}$ より $\cos^2\alpha=\frac{1}{\tan^2\alpha+1}=\frac{1}{(-2)^2+1}=\frac{1}{5}$. α 第 2 象限の角より $\cos\alpha<0$.

$\therefore\cos\alpha=-\frac{1}{\sqrt{5}}$. $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$ より $-2=\frac{\sin\alpha}{-\frac{1}{\sqrt{5}}}$. よって $\sin\alpha=\frac{2}{\sqrt{5}}$

(1) $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\dfrac{2}{\sqrt{5}}\cdot\left(-\dfrac{1}{\sqrt{5}}\right) = -\dfrac{4}{5}$

(2) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(-\dfrac{1}{\sqrt{5}}\right)^2 - \left(\dfrac{2}{\sqrt{5}}\right)^2 = -\dfrac{3}{5}$

(3) $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\cdot(-2)}{1-(-2)^2} = \dfrac{4}{3}$

(4) $\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2} = \dfrac{1-(-\frac{1}{\sqrt{5}})}{2} = \dfrac{\sqrt{5}+1}{2\sqrt{5}} = \dfrac{5+\sqrt{5}}{10}$. α 第 2 象限より $\dfrac{\pi}{2} < \alpha < \pi$ よって

$\dfrac{\pi}{4} < \dfrac{\alpha}{2} < \dfrac{\pi}{2}$. よって $\sin\dfrac{\alpha}{2} > 0$ で $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{5+\sqrt{5}}{10}}$

(5) $\cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2} = \dfrac{1+(-\frac{1}{\sqrt{5}})}{2} = \dfrac{\sqrt{5}-1}{2\sqrt{5}} = \dfrac{5-\sqrt{5}}{10}$. $\dfrac{\pi}{4} < \dfrac{\alpha}{2} < \dfrac{\pi}{2}$ より $\cos\dfrac{\alpha}{2} > 0$ で

$\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{5-\sqrt{5}}{10}}$

(6) $\tan^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha} = \dfrac{1-(-\frac{1}{\sqrt{5}})}{1+(-\frac{1}{\sqrt{5}})} = \dfrac{\sqrt{5}+1}{\sqrt{5}-1} = \dfrac{(\sqrt{5}+1)^2}{5-1} = \dfrac{(\sqrt{5}+1)^2}{4}$. $\dfrac{\pi}{4} < \dfrac{\alpha}{2} < \dfrac{\pi}{2}$ より

$\tan\dfrac{\alpha}{2} > 0$ で $\tan\dfrac{\alpha}{2} = \dfrac{\sqrt{5}+1}{2}$

85.1 ($\sin(-\theta) = -\sin\theta, \cos(-\theta) = n\cos\theta$ に注意)

(1) $\sin x\cos 2x = \dfrac{1}{2}\{\sin(x+2x)+\sin(x-2x)\} = \dfrac{1}{2}(\sin 3x - \sin x)$

(2) $\cos x\cos 3x = \dfrac{1}{2}\{\cos(x+3x)+\cos(x-3x)\} = \dfrac{1}{2}(\cos 4x + \cos 2x)$

(3) $3\cos 3x\sin 5x = \dfrac{3}{2}\{\sin(3x+5x)-\cos(3x-5x)\} = \dfrac{3}{2}(\sin 8x + \sin 2x)$

(4) $\sin 2x + \sin x = 2\sin\dfrac{2x+x}{2}\cos\dfrac{2x-x}{2} = 2\sin\dfrac{3x}{2}\cos\dfrac{x}{2}$

(5) $\sin x - \sin 3x = 2\cos\dfrac{x+3x}{2}\sin\dfrac{x-3x}{2} = 2\cos 2x\sin(-x) = -2\cos 2x\sin x$

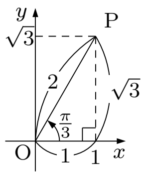
(6) $2(\cos 2x - \cos 6x) = 2(-2\sin\dfrac{2x+6x}{2}\sin\dfrac{2x-6x}{2}) = -4\sin 4x\sin(-2x) = 4\sin 4x\sin 2x$

85.2 (1) $\sin\dfrac{5\pi}{12}\sin\dfrac{\pi}{12} = -\dfrac{1}{2}\left\{\cos\left(\dfrac{5\pi}{12}+\dfrac{\pi}{12}\right)-\cos\left(\dfrac{5\pi}{12}-\dfrac{\pi}{12}\right)\right\} = -\dfrac{1}{2}\left(\cos\dfrac{\pi}{2}-\cos\dfrac{\pi}{3}\right)$

$= -\dfrac{1}{2}\left(0-\dfrac{1}{2}\right) = \dfrac{1}{4}$

(2) $\cos\dfrac{\pi}{12} - \cos\dfrac{5\pi}{12} = -2\sin\dfrac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\sin\dfrac{\frac{\pi}{12}-\frac{5\pi}{12}}{2} = -2\sin\dfrac{\pi}{4}\sin\left(-\dfrac{\pi}{6}\right) = -2\cdot\dfrac{1}{\sqrt{2}}\cdot\left(-\dfrac{1}{2}\right) = \dfrac{1}{\sqrt{2}}$

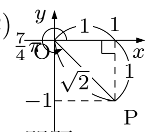
86.1 (1)



OP$=\sqrt{1^2+(\sqrt{3})^2} = 2$　3 辺の比は $2:1:\sqrt{3}$ だから

OP のつくる角度は $60^\circ = \dfrac{\pi}{3}$ よって $\sin x + \sqrt{3}\cos x = 2\sin\left(x+\dfrac{\pi}{3}\right)$

(2)

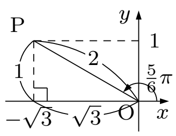


OP$=\sqrt{1^2+(-1)^2} = \sqrt{2}$　3 辺の比は $1:1:\sqrt{2}$ だから

OP のつくる角度は $315^\circ = \dfrac{7}{4}\pi$ よって $y = \sin x - \cos x = \sqrt{2}\sin\left(x+\dfrac{7}{4}\pi\right)$

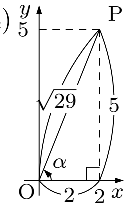
問題に $r\sin(x+\alpha)$ とあるので上記の解答になる．$\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)$ でもよい．

(3)



OP$=\sqrt{(-\sqrt{3})^2+1^2} = 2$　3 辺の比は $2:1:\sqrt{3}$ だから

OP のつくる角度は $150^\circ = \dfrac{5}{6}\pi$ よって $-\sqrt{3}\sin x + \cos x = 2\sin\left(x+\dfrac{5}{6}\pi\right)$

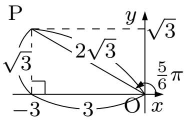
(4) 

OP$=\sqrt{2^2+5^2}=\sqrt{29}$　3 辺の比は $2:1:\sqrt{3}$ だから

OP のつくる角度 α は $\cos\alpha=\dfrac{2}{\sqrt{29}}$, $\sin\alpha=\dfrac{5}{\sqrt{29}}$

$2\sin x+5\cos x=\sqrt{29}\sin(x+\alpha)$

86.2　　OP$=\sqrt{(-3)^2+\sqrt{3}^2}=2\sqrt{3}$　3 辺の比は $2:1:\sqrt{3}$ だから OP のつくる角度は $150^\circ=\dfrac{5}{6}\pi$



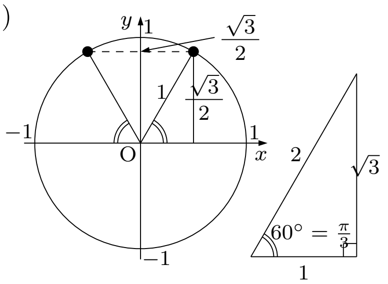
よって $-3\sin x+\sqrt{3}\cos x=2\sqrt{3}\sin\left(x+\dfrac{5}{6}\pi\right)$　$-1\leqq\sin\left(x+\dfrac{5}{6}\pi\right)\leqq 1$ より

最大値 $2\sqrt{3}$, 最小値 $-2\sqrt{3}$　$0\leqq x<2\pi$ より $\dfrac{5}{6}\pi\leqq x+\dfrac{5}{6}\pi<2\pi+\dfrac{5}{6}\pi$ よって

最大値をとるのは $x+\dfrac{5}{6}\pi=\dfrac{\pi}{2}+2\pi\left(\neq\dfrac{\pi}{2}\right)$ よって $x=\dfrac{\pi}{2}+2\pi-\dfrac{5}{6}\pi=\dfrac{5}{3}\pi$ のとき

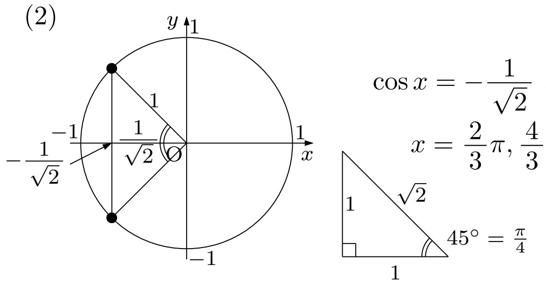
最小値をとるのは $x+\dfrac{5}{6}\pi=\dfrac{3}{2}\pi$ よって $x=\dfrac{3}{2}\pi-\dfrac{5}{6}\pi=\dfrac{2}{3}\pi$ のとき

87.1 (1)



$x=\dfrac{\pi}{3},\dfrac{5}{3}\pi$

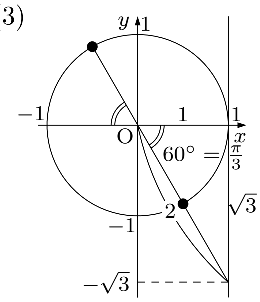
(2)



$\cos x=-\dfrac{1}{\sqrt{2}}$ よって

$x=\dfrac{2}{3}\pi,\dfrac{4}{3}\pi$

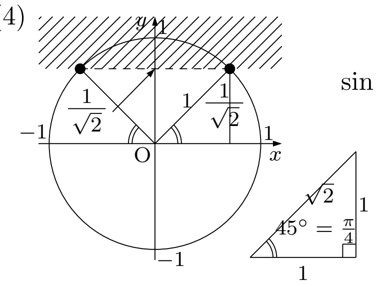
(3)



$\tan x=-\sqrt{3}$ よって

$x=\dfrac{2}{3}\pi,\dfrac{5}{3}\pi$

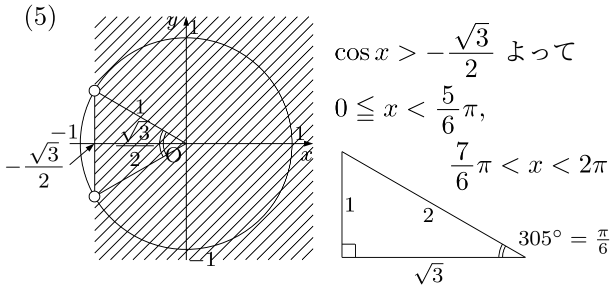
(4)



$\sin x\geqq\dfrac{1}{\sqrt{2}}$ よって

$\dfrac{\pi}{4}\leqq x\leqq\dfrac{3}{4}\pi$

(5)

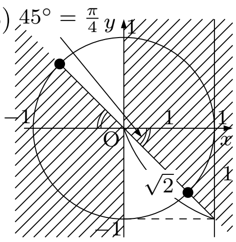


$\cos x>-\dfrac{\sqrt{3}}{2}$ よって

$0\leqq x<\dfrac{5}{6}\pi$,

$\dfrac{7}{6}\pi<x<2\pi$

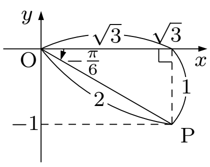
(6)



$0\leqq x<\dfrac{\pi}{2}$, $\dfrac{3}{4}\pi\leqq x<\dfrac{3}{2}\pi$,

$\dfrac{7}{4}\pi\leqq x<2\pi$

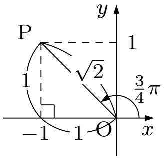
87.2 (1)　　OP$=\sqrt{\sqrt{3}^2+(-1)^2}=2$　3 辺の比は $2:1:\sqrt{3}$ だから OP のつくる角度は $-30^\circ=-\dfrac{\pi}{6}$



よって $\sqrt{3}\sin x-\cos x=2\sin\left(x-\dfrac{\pi}{6}\right)=1$ より $\sin\left(x-\dfrac{\pi}{6}\right)=\dfrac{1}{2}$

$-\dfrac{\pi}{6}\leqq x-\dfrac{\pi}{6}<2\pi-\dfrac{\pi}{6}$ より $x-\dfrac{\pi}{6}=\dfrac{\pi}{6},\dfrac{5}{6}\pi$ よって $x=\dfrac{\pi}{3},\pi$

(2)　　OP$=\sqrt{1^2+(-1)^2}=\sqrt{2}$　3 辺の比は $1:1:\sqrt{2}$ だから OP のつくる角度は $135^\circ=\dfrac{3}{4}\pi$ よって



$\cos x-\sin x=-\sin x+\cos x=\sqrt{2}\sin\left(x+\dfrac{3}{4}\pi\right)=1$ より $\sin\left(x+\dfrac{3}{4}\pi\right)=\dfrac{1}{\sqrt{2}}$

$\dfrac{3}{4}\pi\leqq x+\dfrac{3}{4}\pi<2\pi+\dfrac{3}{4}\pi$ より $x+\dfrac{3}{4}\pi=\dfrac{3}{4}\pi,\dfrac{\pi}{4}+2\pi$ ($\dfrac{\pi}{4}$ は範囲外だから)

よって $x=0,\dfrac{3}{2}\pi$