# 第1章 基礎解析

## §1. 式の計算

1.1 (1) $\dfrac{4(x+2)}{(x+1)^2(x+3)} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x+3}$

$4x+8 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$

$x=-1$ を代入して $4 = 2B$ $B=2$

$x=-3$ を代入して $-4 = 4C$ $C=-1$

$x=-2$ を代入して $0 = -A+B+C$ $A=1$

$\therefore \dfrac{4(x+2)}{(x+1)^2(x+3)} = \dfrac{1}{x+1} + \dfrac{2}{(x+1)^2} - \dfrac{1}{x+3}$

(2) $\dfrac{4x^2}{x^4-1} = \dfrac{2}{x^2-1} + \dfrac{2}{x^2+1} = \dfrac{1}{x-1} - \dfrac{1}{x+1} + \dfrac{2}{x^2+1}$

(3) $\dfrac{1}{x(x+1)(x+2)\cdots(x+n)} = \dfrac{a_0}{x} + \dfrac{a_1}{x+1} + \dfrac{a_2}{x+2} + \cdots + \dfrac{a_k}{x+k} + \cdots + \dfrac{a_n}{x+n}$

$\therefore 1 = a_0(x+1)(x+2)\cdots(x+n) + a_1 x(x+2)\cdots(x+n) + \cdots + a_k x(x+1)\cdots(x+k-1)(x+k+1)\cdots(x+n)$
$+ \cdots + a_n x(x+1)\cdots(x+n-1)$

$x=-k$ $k=0, 1, 2, \cdots n$ を代入して

$1 = (-k)(-k+1)\cdots(-1)\cdot 1\cdot 2\cdots(n-k)a_k$

$\therefore a_k = \dfrac{(-1)^k}{k!(n-k)!}$

$\therefore \dfrac{1}{x(x+1)(x+2)\cdots(x+n)} = \displaystyle\sum_{k=0}^{n} \dfrac{(-1)^k}{k!(n-k)!(x+k)}$

1.2 $\dfrac{y+z}{x} = \dfrac{z+7x}{y} = \dfrac{x-y}{z} = k$ とおくと

$y+z = kx,\ z+7x = ky,\ x-y = kz$

$y-7x = k(x-y)$ $(1+k)y = (k+7)x$ $\therefore y = \dfrac{k+7}{1+k}x$

$z = kx - \dfrac{k+7}{k+1}x = \dfrac{k^2-7}{k+1}x$ これを第3式に代入して

$x - \dfrac{k+7}{k+1}x = \dfrac{k(k^2-7)}{k+1}x$ $\therefore -6x = (k^3-7k)x$ $x \neq 0$

$k^3-7k+6=0$

$(k-1)(k-2)(k+3)=0$

$\therefore k = 1, 2, -3$

$$\begin{array}{rrrr|l}
1 & 0 & -7 & 6 & 1 \\
 & 1 & 1 & -6 & \\
\hline
1 & 1 & -6 & 0 & \\
 & 2 & 6 & & 2 \\
\hline
1 & 3 & 0 & &
\end{array}$$

1.3 $x^4+3x^2+ax^2+bx+c=\{(x+\lambda_1)x+\lambda_2\}\{(x+\lambda_1)x+\lambda_3\}+\lambda_4$

$$=(x+\lambda_1)^2x^2+(\lambda_2+\lambda_3)(x+\lambda_1)x+\lambda_2\lambda_3+\lambda_4$$

$$=x^4+2\lambda_1x^3+(\lambda_1^2+\lambda_2+\lambda_3)x^2+(\lambda_2+\lambda_3)\lambda_1x+\lambda_2\lambda_3+\lambda_4$$

$3=2\lambda_1$, $a=\lambda_1^2+\lambda_2+\lambda_3$ $b=\lambda_1(\lambda_2+\lambda_3)$, $c=\lambda_2\lambda_3+\lambda_4$

$\lambda_1=\frac{3}{2}$ // $\lambda_2+\lambda_3=a-\frac{9}{4}$. $\lambda_2+\lambda_3=\frac{2}{3}b$

$\therefore$ $a-\frac{9}{4}=\frac{2}{3}b$ $12a-8b=27$ のとき

$\lambda_2=\frac{2}{3}b-\lambda$, $\lambda_3=\lambda$ $\cdot$ $\lambda_4=c-\frac{2}{3}b\lambda+\lambda^2$ //

$12a-8b\neq27$ のとき 解なし

1.4 $f(x)=(x-\alpha)(x^2-\beta)g(x)+Ax^2+Bx+C$ とおく

$f(\alpha)=m$, $f(\sqrt{\beta})=p\sqrt{\beta}+q$, $f(-\sqrt{\beta})=-p\sqrt{\beta}+q$ より

$m=A\alpha^2+B\alpha+C$ $A\beta+B\sqrt{\beta}+C=p\sqrt{\beta}+q$

$A\beta-B\sqrt{\beta}+C=-p\sqrt{\beta}+q$

$\therefore$ $B\sqrt{\beta}=p\sqrt{\beta}$ $\therefore$ $B=p$ $\therefore$ $A\beta+C=q$

$\therefore$ $m=A\alpha^2+p\alpha+q-A\beta$ $\therefore$ $m-p\alpha-q=A(\alpha^2-\beta)$

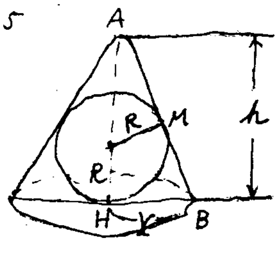
$\alpha^2\neq\beta$ のとき $A=\frac{m-p\alpha-q}{\alpha^2-\beta}$. $B=p$, $C=q-\frac{m\beta-p\alpha\beta-q\beta}{\alpha^2-\beta}$

$$=\frac{q\alpha^2-m\beta+p\alpha\beta}{\alpha^2-\beta}$$

$\alpha^2=\beta$, $m=p\alpha+q$ のとき $A=a$ (任意) $B=p$, $C=q-\beta a$

$\alpha^2=\beta$ $m\neq p\alpha+q$ のとき 解なし

1.5




$AB=\sqrt{r^2+h^2}$

$(\sqrt{r^2+h^2}-r)^2+R^2=(h-R)^2$

$2r^2-2r\sqrt{r^2+h^2}=-2hR$

$R=\frac{r(\sqrt{r^2+h^2}-r)}{h}$

## §2. 方程式

2.1 $x^4-14x^3+74x^2-182x+169=0$. の根を $\alpha.\beta.\gamma.\delta$ とすると

$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)=0$

$\{x^2-(\alpha+\beta)x+\alpha\beta\}\{x^2-(\gamma+\delta)x+\gamma\delta\}=0$

$\alpha\beta=\gamma\delta=\pm\sqrt{169}=\pm 13$

$\alpha+\beta+\gamma+\delta=14 \qquad \alpha\beta+\gamma\delta+(\alpha+\beta)(\gamma+\delta)=74$

$\therefore\ 2\alpha\beta+(\alpha+\beta)(\gamma+\delta)=74 \qquad (\alpha+\beta+\gamma+\delta)\alpha\beta=182$

$\therefore\ \alpha\beta=\dfrac{182}{14}=13$

$\therefore\ (\alpha+\beta)+(\gamma+\delta)=14. \quad (\alpha+\beta)(\gamma+\delta)=48$

$\alpha+\beta=6,\ \gamma+\delta=8$ or $\alpha+\beta=8,\ \gamma+\delta=6$.

$\therefore\ (x^2-6x+13)(x^2-8x+13)=0$

$\therefore\ x=3\pm 2i \ ,\ 4\pm\sqrt{3}$

2.2 $x^3-1=0\ , \qquad ax^2+bx+c=0$

$(x-1)(x^2+x+1)=0 \quad \therefore\ x=1.\ \dfrac{-1\pm\sqrt{3}i}{2} \qquad \omega=\dfrac{-1+\sqrt{3}i}{2}$ とおくと

$\omega^2=\dfrac{-1-\sqrt{3}i}{2} \qquad \omega^3=1. \quad a\omega^2+b\omega+c=0 \quad a\omega+b\omega^2+c=0$

$$a^3+b^3+c^3-3abc=\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}=\begin{vmatrix} a+c\omega+b\omega^2 & b+a\omega+c\omega^2 & c+b\omega+a\omega^2 \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$=\begin{vmatrix} \omega(a\omega^2+c+b\omega) & \omega^2(a\omega^2+b\omega+c) & a\omega^2+b\omega+c \\ c & a & b \\ b & c & a \end{vmatrix}=0$$

2.3 $x^3+px^2+qx+r=0$ の 3 根 $\alpha.\beta.\gamma$

$\alpha+\beta+\gamma=-p \quad \alpha\beta+\beta\gamma+\gamma\alpha=q \quad \alpha\beta\gamma=-r$

$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)=p^2-2q$

$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\beta+\gamma+\alpha)=q^2-2pr$

(1) $(x-\alpha^2)(x-\beta^2)(x-\gamma^2)=0$

$x^3-(\alpha^2+\beta^2+\gamma^2)x^2+(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)x-\alpha^2\beta^2\gamma^2=0$

$x^3-(p^2-2q)x^2+(q^2-2pr)x-r^2=0$

(2) $(x-\alpha\beta)(x-\beta\gamma)(x-\gamma\alpha)=0$

$$x^3-(\alpha\beta+\beta\gamma+\gamma\alpha)x^2+\alpha\beta\gamma(\beta+\gamma+\alpha)x-\alpha^2\beta^2\gamma^2=0$$

$$x^3-qx^2+prx-r^2=0$$

2.4
$$\begin{cases} x-2y+3z=2 & \cdots ① \\ 2x-3y+4z=3 & \cdots ② \\ 3x-8y+13z=8 & \cdots ③ \end{cases}$$

②$-2\times$① $y-2z=-1$

③$-3\times$① $-2y+4z=2$

$y-2z=-1$

∴ $z=t$ とおくと $y=2t-1$ $x=t$ $t$ は任意の数

2.5 $x^3+px^2+qx+r=0$ ($p.q.r$ は整数) の解を $\alpha.\beta.\gamma$ とする

$a_n=\alpha^n+\beta^n+\gamma^n$ は整数であること

[I] $n=1$ のとき $a_1=\alpha+\beta+\gamma=-p$ 整数

$n=2$ のとき $a_2=\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=p^2-2q$ 整数

$n=3$ のとき $a_3=\alpha^3+\beta^3+\gamma^3=(\alpha+\beta)^3+\gamma^3-3\alpha\beta(\alpha+\beta)$

$=(\alpha+\beta+\gamma)\{(\alpha+\beta)^2-(\alpha+\beta)\gamma+\gamma^2\}-3\alpha\beta(\alpha+\beta+\gamma)+3\alpha\beta\gamma$

$=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)+3\alpha\beta\gamma$

$=-p(p^2-2q-q)-3r=-p^3+3pq-3r$ 整数

[II] $n=k.\ k-1\ \ k-2$ のとき $a_n$ は整数とすると

$n=k+1$ のとき

$a_{k+1}=\alpha^{k+1}+\beta^{k+1}+\gamma^{k+1}=(\alpha+\beta+\gamma)(\alpha^k+\beta^k+\gamma^k)-\alpha(\beta^k+\gamma^k)-\beta(\alpha^k+\gamma^k)-\gamma(\alpha^k+\beta^k)$

$=-pa_k-\alpha\beta(\beta^{k-1}+\alpha^{k-1})-\beta\gamma(\beta^{k-1}+\gamma^{k-1})-\gamma\alpha(\gamma^{k-1}+\alpha^{k-1})$

$=-pa_k-\alpha\beta(a_{k-1}-\gamma^{k-1})-\beta\gamma(a_{k-1}-\alpha^{k-1})-\gamma\alpha(a_{k-1}-\beta^{k-1})$

$=-pa_k-a_{k-1}(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma(\alpha^{k-2}+\beta^{k-2}+\gamma^{k-2})$

$=-pa_k-qa_{k-1}-ra_{k-2}$ 整数

∴ [I] [II] より 自然数 $n$ に対して $a_n$ は整数

2.6 $z=3=3(\cos 2k\pi+i\sin 2k\pi)$ $k=0.1\ 2$

$z^{\frac{1}{3}}=3^{\frac{1}{3}}(\cos\frac{2k}{3}\pi+i\sin\frac{2k}{3}\pi)$

∴ $\sqrt[3]{3}$, $\sqrt[3]{3}\,\omega$, $\sqrt[3]{3}\,\omega^2$ $\omega=\frac{1}{2}(-1+\sqrt{3}i)$

# §3 三角函数, 对数函数

3.1 $\cos\theta = x$. $\cos 4\theta = 2\cos^2 2\theta - 1 = 2(2\cos^2\theta - 1)^2 - 1$

$= 2(2x^2-1)^2 - 1 = 8x^4 - 8x^2 + 1$

3.2 $\sin 3x + \sin(x + \frac{\pi}{2}) = \sqrt{3}\sin(x + \frac{\pi}{4})$

$2\sin(2x + \frac{\pi}{4})\cos(x - \frac{\pi}{4}) = \sqrt{3}\sin(x + \frac{\pi}{4})$

$2\sin(2x + \frac{\pi}{4})\sin(x + \frac{\pi}{4}) = \sqrt{3}\sin(x + \frac{\pi}{4})$

$(2\sin(2x + \frac{\pi}{4}) - \sqrt{3})\sin(x + \frac{\pi}{4}) = 0$

$\therefore \sin(2x + \frac{\pi}{4}) = \frac{\sqrt{3}}{2}$ $\quad \sin(x + \frac{\pi}{4}) = 0$

$2x + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{3}$ $\quad x + \frac{\pi}{4} = n\pi$

$x = \frac{n}{2}\pi + (-1)^n\frac{\pi}{6} - \frac{\pi}{8}$, $\quad x = n\pi - \frac{\pi}{4}$

3.3 $\alpha + \beta = \gamma$ $\quad 0 < |\gamma| < \pi$. $\quad \beta = \gamma - \alpha$

$\sin\alpha + \sin\beta = \sin\alpha + \sin(\gamma - \alpha)$

$= 2\sin\frac{\gamma}{2}\cos(\alpha - \frac{\gamma}{2})$

i) $0 < \gamma < \pi$ [illegible] $2\sin\frac{\gamma}{2}$. $\quad \alpha = \frac{\gamma}{2} + 2n\pi$ $\quad \beta = \frac{\gamma}{2} - 2n\pi$

ii) $-\pi < \gamma < 0$ ,, $-2\sin\frac{\gamma}{2}$ $\quad \alpha = \frac{\gamma}{2} + (2n+1)\pi$ $\quad \beta = \frac{\gamma}{2} - (2n+1)\pi$

3.4 $\cos(x + \frac{2}{3}\pi) + \sin(x + \frac{1}{4}\pi)$

$= \cos x\cos\frac{2}{3}\pi - \sin x\sin\frac{2}{3}\pi + \sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4}$

$= \cos x\,(-\frac{1}{2} + \frac{\sqrt{2}}{2}) + \sin x\,(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2})$

$= \frac{1}{2}(\sqrt{2} - 1)\cos x + \frac{1}{2}(\sqrt{2} - \sqrt{3})\sin x$

3.5 $25x^2 - kx + 12 = 0$ の 二根 $\sin 2\theta$, $\cos 2\theta$

(1) $\sin 2\theta + \cos 2\theta = \frac{k}{25}$ $\quad \sin 2\theta\cos 2\theta = \frac{12}{25}$

$1 + 2\sin 2\theta\cos 2\theta = \frac{k^2}{25^2}$

$\therefore \frac{k^2}{25^2} - 1 - \frac{24}{25} = 0$ $\quad \frac{k}{25} = \pm\frac{7}{5}$ $\quad k = \pm 35$

(2) $25x^2 \mp 35x + 12 = 0$ $\quad (5x \mp 3)(5x \mp 4) = 0$

$x = \frac{3}{5}$ $\frac{4}{5}$, $-\frac{3}{5}$, $-\frac{4}{5}$

$\tan 2\theta = \frac{3}{4}$ or $\frac{4}{3}$

3.2

(3) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ $\qquad \tan 2\theta(1-\tan^2\theta) = 2\tan\theta$

$\tan 2\theta = \dfrac{3}{4}$ のとき $\qquad 3(1-\tan^2\theta) = 8\tan\theta$

$3\tan^2\theta + 8\tan\theta - 3 = 0 \qquad (3\tan\theta - 1)(\tan\theta + 3) = 0$

$\tan\theta = \dfrac{1}{3},\ -3$ //

$\tan 2\theta = \dfrac{4}{3}$ のとき $\qquad 4\tan^2\theta + 6\tan\theta - 4 = 0$

$(2\tan\theta - 1)(\tan\theta + 2) = 0$

$\tan\theta = \dfrac{1}{2},\ -2$ //

3.6 $3\sin x + 4\cos x = a\cos(x-\phi)$

$5\left(\dfrac{3}{5}\sin x + \dfrac{4}{5}\cos x\right) = a\cos(x-\phi)$ $\qquad \sin\alpha = \dfrac{3}{5}\quad \cos\alpha = \dfrac{4}{5}$ とおく

$5\cos(x-\alpha) = a\cos(x-\phi)$

$\therefore\ a = 5 \quad \phi = \alpha$

$\therefore\ a = 5 \quad \tan\phi = \dfrac{3}{4}$

3.7 $a.\ b > 0 \qquad a^a < b < 1$

(1) $a < b^{\frac{1}{a}} < 1^{\frac{1}{a}} = 1 \qquad \therefore\ a < 1$

(2) (1)より $a < 1$ $\therefore$ $\quad a > \log_a b > 0$

$\therefore\ 0 < \log_a b < 1$

(3) $b^{\frac{1}{a}} > a > \log_a b$

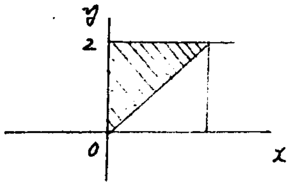
$\dfrac{1}{a}\log_a b < \log_a(\log_a b)$

$\therefore\ \log_a b < a\log_a(\log_a b) < \log_a(\log_a b)$

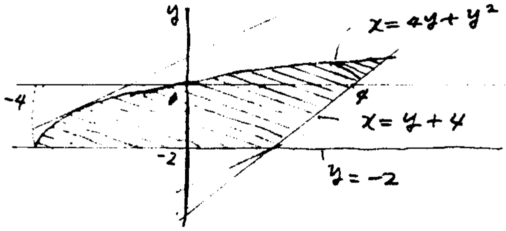
$\therefore\ \log_a b < \log_a(\log_a b).$

# §4 領域

4.1




4.2




$2y-x=k$ とおくと　$y=\frac{1}{2}x+\frac{k}{2}$

$2y-k=4y+y^2$　$y^2+2y+k=0$　$k=1$　$y=-1,\ x=-3$

最大値 $f(-3,-1)=-2+3=1$

最小値 $f(2,-2)=-4-2=-6$

# §5 場合の数

5.1 (1) $2+2(n-1)\leqq 100$　$n\leqq 50$　50個

(2) $6+6(n-1)\leqq 100$　$n\leqq \frac{100}{6}$　16

$50+33-16=67$ 個

(3) $100-(50+33+20-16-10-6+3)=26$ 個

5.2 □□□□□□□　1, 1, 2, 2, 3, 3, 4

$\frac{4!}{2!2!}\times\frac{3!}{2!}=18$

5.3 a a a b b c c d e f

a 3個 他 1個　$5\times\frac{4!}{3!}=20$

a, b, c から2個ずつ　${}_3C_2\,\frac{4!}{2!2!}=3\times 6=18$

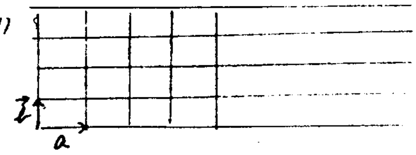
a, b, c から2個他から2個　${}_3C_1\,{}_5C_2\cdot\frac{4!}{2!}=360$

各文字から1個　${}_6C_4\cdot 4!=15\cdot 24=360$

758

5.4

(1)




$f(1,1)=2 \quad f(2,1)=f(1,2)=3$

$f(2,2)=6$

$f(7,7)=\dfrac{14!}{7!\cdot 7!}=3432$

(2) $f(i,j)\times f(m-i,\ n-j)$ $\qquad f(0,0)=f(1,0)=f(0,1)=1$ とする。

(3) $f(i,j)\,f(m-i,\ n-j)+f(k,l)\cdot f(m-k,\ n-l)-f(i,j)\,f(k-i,\ l-j)\,f(m-k,\ n-l)$

5.5 2ビット 00, 01, 10, 11,

3ビット $2^3$ 4ビット $2^4$ … 10ビット $2^{10}$

8, 16, … 1024,

1～10まで 4ビット $26 < 32 = 2^5$ 5ビット

5.6 $3^{10}-3\times 2^{10}+3=55980$

5.7 白 $n$ 個 他 0 個 ${}_nC_0$ 白 $n-1$ 個 他 1 個 ${}_nC_1$ 白 $n-k$ 個 他 $k$ 個 ${}_nC_k$

$\therefore {}_nC_0+{}_nC_1+{}_nC_2+ \quad +{}_nC_n=(1+1)^n=2^n$

5.8 $\dfrac{8!}{3!5!}=56$

# §6 二項定理

6.1 (1) $\sum_{m=0}^{n} {}_nC_m x^m(1-x)^{n-m} = \{x+(1-x)\}^n = 1$

(2) $\sum_{m=1}^{n} \frac{m}{n} {}_nC_m x^m(1-x)^{n-m} = \sum_{m=1}^{n} \frac{m}{n} \frac{n!}{m!(n-m)!} x^m(1-x)^{n-m}$

$= \sum_{m=1}^{n} \frac{(n-1)!}{(m-1)!(n-m)!} x^m(1-x)^{n-m} = x\sum_{m=0}^{n-1} {}_{n-1}C_m x^m(1-x)^{n-1-m}$

$m-1 \to m$ とおく

$= x$

(3) $\sum_{m=1}^{n} \frac{m^2}{n^2} {}_nC_m x^m(1-x)^{n-m} = \sum_{m=1}^{n} \frac{m^2}{n^2} \frac{n!}{m!(n-m)!} x^m(1-x)^{n-m}$

$= \sum_{m=1}^{n} \frac{m}{n} \frac{(n-1)!}{(m-1)!(n-m)!} x^m(1-x)^{n-m}$

$= \frac{x}{n} \sum_{k=0}^{n-1} (k+1)\,{}_{n-1}C_k\, x^k(1-x)^{n-1-k}$ $\qquad m-1=k$ とおく

$\frac{d}{dx} x(x+y)^{n-1} = \sum_{k=0}^{n-1} \frac{d}{dx} {}_{n-1}C_k\, x^{k+1}y^{n-1-k} = \sum_{k=0}^{n-1} (k+1)\,{}_{n-1}C_k\, x^k y^{n-1-k}$

$\therefore (x+y)^{n-1} + (n-1)x(x+y)^{n-2} = \sum_{k=0}^{n-1} (k+1)\,{}_{n-1}C_k\, x^k y^{n-1-k}$

$y=1-x$ を代入して

$1+(n-1)x = \sum_{k=0}^{n-1} (k+1)\,{}_{n-1}C_k\, x^k(1-x)^{n-1-k}$

$\therefore \sum_{m=1}^{n} \frac{m^2}{n^2} {}_nC_k x^k(1-x)^{n-k} = \frac{x}{n}\{1+(n-1)x\}$

6.2 (1) (a) ${}_nC_k = \frac{n!}{k!(n-k)!} = {}_nC_{n-k}$

(b) ${}_nC_k + {}_nC_{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n+1-k)!}$

$= \frac{(n+1-k+k)n!}{k!(n+1-k)!} = \frac{(n+1)!}{k!(n+1-k)!} = {}_{n+1}C_k$

(c) $(1+x)^n = \sum_{k=0}^{n} {}_nC_k x^k$

$x=-1$ を代入して

$0 = \sum_{k=0}^{n} (-1)^k {}_nC_k$

(2) (a) $\sum_{k=0}^{2n}(-1)^k {}_{2n}C_k = 0$  ${}_{2n}C_{2n-k} = {}_{2n}C_k$ より

$$\sum_{k=0}^{n}(-1)^k {}_{2n}C_k + \sum_{k=n+1}^{2n}(-1)^k {}_{2n}C_k = 0$$

$$\sum_{k=0}^{n}(-1)^k {}_{2n}C_k + \sum_{k=n+1}^{2n}(-1)^k {}_{2n}C_{2n-k} = 0$$ $2n-k=h$

$k=2n-h$ $(-1)^k=(-1)^h$

$$\sum_{k=0}^{n}(-1)^k {}_{2n}C_k + \sum_{h=0}^{n-1}(-1)^h {}_{2n}C_h = 0$$

$$(-1)^n {}_{2n}C_n = 2\sum_{k=0}^{n}(-1)^k {}_{2n}C_k$$

$$\therefore \sum_{k=0}^{n}(-1)^k {}_{2n}C_k = \frac{1}{2}(-1)^n {}_{2n}C_n$$

(b) $\sum_{k=0}^{n}(-1)^k {}_{2n+1}C_k$  ${}_{2n+1}C_k = {}_{2n}C_k + {}_{2n}C_{k-1}$ より

$$= {}_{2n+1}C_0 + \sum_{k=1}^{n}(-1)^k {}_{2n+1}C_k = 1 + \sum_{k=1}^{n}(-1)^k\{{}_{2n}C_k + {}_{2n}C_{k-1}\}$$

$$= 1 + \sum_{k=1}^{n}(-1)^k {}_{2n}C_k - \sum_{k=1}^{n}(-1)^{k-1} {}_{2n}C_{k-1}$$

$$= \sum_{k=0}^{n}(-1)^k {}_{2n}C_k - \sum_{k=0}^{n-1}(-1)^k {}_{2n}C_k = (-1)^n {}_{2n}C_n$$

$$\therefore \sum_{k=0}^{n}(-1)^k {}_{2n+1}C_k = (-1)^n {}_{2n}C_n$$

6.3 $(2x^3+3x^{-2})^5$ の一般項 ${}_5C_r(2x^3)^r(3x^{-2})^{5-r}$

$x^{3r}\cdot x^{-10+2r} = x^{-10+5r}$  $-10+5r=0$  $r=2$

$\therefore {}_5C_2\ 2^2\cdot 3^3 = 10\cdot 4\cdot 27 = 1080$

6.4 $(1+x)^{12}$  一般項 ${}_{12}C_r\, x^{12-r}$

$12-r=10$  $r=2$  ${}_{12}C_2 = \frac{12\cdot 11}{2} = 66$