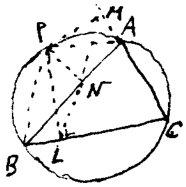
第2章 解析幾何

§1 平面図形

1.1



Pは $\widehat{AB}$ 上にあるとすると

AP.BCは同一円周上にあるから

$\angle ACB + \angle APB = 2\angle R$

PMCLは同一円周上にあるから

$\angle ACB + \angle MPL = 2\angle R$

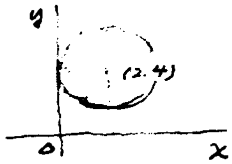
∴ $\angle APB = \angle MPL$　∴ $\angle BPL = \angle APM$

PNLBは同一円周上にあるから $\angle BPL = \angle BNL$

PMAN 〃　$\angle APM = \angle MNA$

∴ $\angle BNL = \angle MNA$　∴ L.M.Nは一直線にある。

1.2 (1)



$(x-2)^2+(y-4)^2=4$

(2) $y=x+k$ とおくと

$(x-2)^2+(x+k-4)^2=4$

$2x^2+2x(k-6)+(k-4)^2=0$

$\frac{D}{4}=(k-6)^2-2(k-4)^2$　$-k^2+4k+4=0$

$k=2\pm\sqrt{4+4}=2\pm2\sqrt{2}$

∴ $y=x+2(1\pm\sqrt{2})$

(3) $\begin{cases}(x-2)^2+(y-4)^2=4\\ x^2+y^2=r^2\end{cases}$　$2x+\lambda\cdot2(x-2)=0$　$x(1+\lambda)=2\lambda$

$2y+\lambda\cdot2(y-4)=0$　$y(1+\lambda)=4\lambda$

$\frac{x^2}{\lambda^2}+\frac{y^2}{\lambda^2}=4$　$\frac{4}{(1+\lambda)^2}+\frac{16}{(1+\lambda)^2}=4$　$5=(1+\lambda)^2$　$\lambda=-1\pm\sqrt{5}$

$1+\lambda=\pm\sqrt{5}$　$x=\frac{-2\pm2\sqrt{5}}{\pm\sqrt{5}}=2\mp\frac{2}{\sqrt{5}}$　$y=4\mp\frac{4}{\sqrt{5}}$

$x^2+y^2=4\lambda^2=4(-1\pm\sqrt{5})^2=4(6\mp2\sqrt{5})=\{2(\sqrt{5}\pm1)\}^2$

∴ $x=2+\frac{2}{\sqrt{5}}$, $y=4+\frac{4}{\sqrt{5}}$ のとき 最大値 $2(\sqrt{5}+1)$

(条件付極大.極小)

1.3 (I) ; $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(II) : $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(1-3) & \frac{1}{\sqrt{2}}(1-2) \\ \frac{1}{\sqrt{2}}(1+3) & \frac{1}{\sqrt{2}}(1+2) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}}(-2x-y) \\ \frac{1}{\sqrt{2}}(4x+3y) \end{pmatrix} \qquad \therefore x'' = -\frac{1}{\sqrt{2}}(2x+y)$$

$$y'' = \frac{1}{\sqrt{2}}(4x+3y)$$

$y'' = ax'' + b$ に代入して

$4x + 3y = -a(2x+y) + b\sqrt{2}$

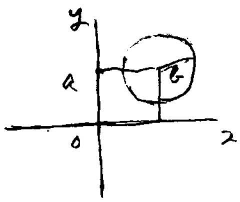
$(3+a)y = (-2a-4)x + b\sqrt{2}$ $\qquad \therefore y = \frac{-2(a+2)}{3+a}x + b\frac{\sqrt{2}}{3+a}$

$\therefore a = \frac{-2(a+2)}{3+a} \qquad b = b\frac{\sqrt{2}}{3+a}$

$a^2 + 3a + 2a + 4 = 0 \qquad a^2 + 5a + 4 = 0$

$a = -1, -4, \quad b = 0$

1.4 $(|x| - a)^2 + (|y| - a)^2 = b^2$



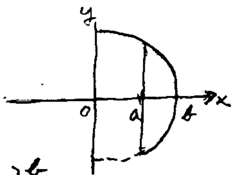
$x \geqq 0, \quad y \geqq 0$ のとき $\quad (x-a)^2 + (y-a)^2 = b^2$

$x < 0 \quad y \geqq 0 \qquad (x+a)^2 + (y-a)^2 = b^2$

$x < 0 \quad y < 0 \qquad (x+a)^2 + (y+a)^2 = b^2$

$x > 0 \quad y < 0 \qquad (x-a)^2 + (y+a)^2 = b^2$

i) $b \leqq a$ のとき $\quad 4\pi b^2$

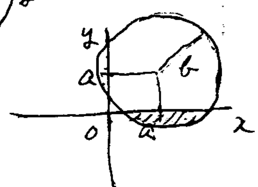


ii) $a \leqq b \leqq \sqrt{2}a$ のとき $\quad x^2 + y^2 = b^2$

$y = \pm\sqrt{b^2 - x^2} \qquad \int_a^b 2\sqrt{b^2 - x^2}\,dx$

$= \left[x\sqrt{b^2 - x^2} + b^2 \sin^{-1}\frac{x}{b}\right]_a^b$

$= \frac{\pi}{2}b^2 - a\sqrt{b^2 - a^2} - b^2 \sin^{-1}\frac{a}{b}$

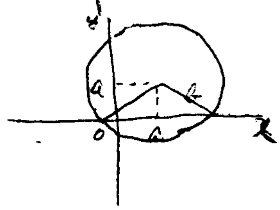


$4\pi b^2 - 8\left(\frac{\pi}{2}b^2 - a\sqrt{b^2 - a^2} - b^2 \sin^{-1}\frac{a}{b}\right)$

$= 8a\sqrt{b^2 - a^2} + 8b^2 \sin^{-1}\frac{a}{b}$

iii) $\sqrt{2}a \leqq b$ のとき $\quad (x-a)^2 + (y-a)^2 = b^2$

$y = a \pm \sqrt{b^2 - (x-a)^2}$



P4

$$\int_0^{a+\sqrt{b^2-a^2}} \left(a+\sqrt{b^2-(x-a)^2}\right)dx + \int_{a+\sqrt{b^2-a^2}}^{a+b} 2\sqrt{b^2-(x-a)^2}\,dx$$

$$= \left[ax + \frac{1}{2}\left\{(x-a)\sqrt{b^2-(x-a)^2} + b^2\sin^{-1}\frac{x-a}{b}\right\}\right]_0^{a+\sqrt{b^2-a^2}}$$

$$+ \left[(x-a)\sqrt{b^2-(x-a)^2} + b^2\sin^{-1}\frac{x-a}{b}\right]_{a+\sqrt{b^2-a^2}}^{a+b}$$

$$= a\left(a+\sqrt{b^2-a^2}\right) + \frac{1}{2}\sqrt{b^2-a^2}\,a + \frac{1}{2}b^2\sin^{-1}\frac{\sqrt{b^2-a^2}}{b} + \frac{1}{2}a\sqrt{b^2-a^2} + \frac{1}{2}b^2\sin^{-1}\frac{a}{b}$$

$$+ b^2\frac{\pi}{2} - \sqrt{b^2-a^2}\,a - b^2\sin^{-1}\frac{\sqrt{b^2-a^2}}{b}$$

$$= a\left(a+\sqrt{b^2-a^2}\right) + \frac{1}{2}b^2\sin^{-1}\frac{a}{b} + \frac{\pi}{2}b^2 - \frac{1}{2}b^2\sin^{-1}\frac{\sqrt{b^2-a^2}}{b}$$

$$\therefore\ 4a\left(a+\sqrt{b^2-a^2}\right) + 2b^2\sin^{-1}\frac{a}{b} + 2\pi b^2 - 2b^2\sin^{-1}\frac{\sqrt{b^2-a^2}}{b}$$

1.5 $x^2+2axy+y^2=1$ 閉曲線

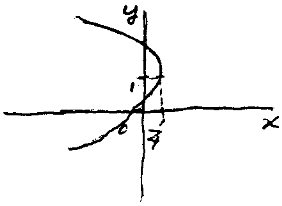
$(x+ay)^2+(1-a^2)y^2=1$

$\therefore\ 1-a^2>0 \quad \therefore\ |a|<1$

1.6 $y^2+4x-2y=0$

$(y-1)^2=-4x+1$

$\therefore\ (y-1)^2=4(-1)(x-\frac{1}{4})$



1.7 $(x-a)^2+(y-b)^2=r^2$ $\quad S(s,s^2)\quad T(t,t^2)\quad U(u,u^2)$

$(s-a)^2+(s^2-b)^2=r^2$ $\qquad t^2-s^2+2a(s-t)+t^4-s^4+2b(s^2-t^2)=0$

$(t-a)^2+(t^2-b)^2=r^2$ $\qquad u^2-s^2+2a(s-u)+u^4-s^4+2b(s^2-u^2)=0$

$(u-a)^2+(u^2-b)^2=r^2$

$-(t+s)+2a-(s+t)(s^2+t^2)+2b(s+t)=0$

$-(u+s)+2a-(s+u)(s^2+u^2)+2b(s+u)=0$

$-(t-u)-(st^2+s^2t+t^3-su^2-s^2u-u^3)+2b(t-u)=0$

$-(t-u)-\{s(t^2-u^2)+s^2(t-u)+(t-u)(t^2+tu+u^2)\}+2b(t-u)=0$

$-1-(st+su+s^2+t^2+tu+u^2)=-2b$

$b=\frac{1}{2}(1+s^2+t^2+u^2+st+tu+su)$

$a=\frac{-1}{2}(s+t)(t+u)(u+s)$

1.8 $2x^2-3xy+\lambda y^2+5y+\mu=0$ ①

xについて解くと xはyの一次式

∴ $9y^2-8(\lambda y^2+5y+\mu)$ は yの一次式の自乗である

$(9-8\lambda)y^2-40y-8\mu=(ay+b)^2$

∴ $20^2+8\mu(9-8\lambda)=0$ $\quad 50+\mu(9-8\lambda)=0$

①より $x=\frac{1}{4}\left(3y\pm\sqrt{(9-8\lambda)y^2-40y-8\mu}\right)$

$(9-8\lambda)y^2-40y-8\mu=\left(\sqrt{9-8\lambda}\,y+\alpha\right)^2$ より

yの係数は $\frac{3+\sqrt{9-8\lambda}}{4}$, $\frac{3-\sqrt{9-8\lambda}}{4}$ 直交するから

∴ $\frac{9-(9-8\lambda)}{16}=-1$ $\quad 8\lambda=-16$ $\quad \lambda=-2,$ $\quad \mu=-2$

∴ $2x^2-3xy-2y^2+5y-2=0$ $\quad 2x^2-3xy-(2y-1)(y-2)=0$

$(x-2y+1)(2x+y-2)=0$

1.9 $2x+y=0$ $\quad \begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}\cos45^\circ & \sin45^\circ\\ -\sin45^\circ & \cos45^\circ\end{pmatrix}\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt2}(X+Y)\\ \frac{1}{\sqrt2}(-X+Y)\end{pmatrix}$

$\sqrt2(X+Y)+\frac{1}{\sqrt2}(-X+Y)=0$ $\quad X+3Y=0$

1.10 $at=b\theta$ $\quad \theta=\frac{a}{b}t$

$x=(a+b)\cos t+b\cos(\pi-t-\theta)$

$\quad =(a+b)\cos t-b\cos(t+\theta)$

$$\begin{cases} x=(a+b)\cos t-b\cos\frac{a+b}{b}t\\ y=(a+b)\sin t-b\sin\frac{a+b}{b}t\end{cases}$$

§2 直線，平面

2.1 平面 $Ax+By+Cz+D=0$ の方向余弦は

(イ) $\left(\dfrac{A}{\alpha}, \dfrac{B}{\alpha}, \dfrac{C}{\alpha}\right)$ $\quad \alpha=\sqrt{A^2+B^2+C^2}$

P_0 を通り平面に垂直な直線は

$$\frac{\alpha(x-x_0)}{A}=\frac{\alpha(y-y_0)}{B}=\frac{\alpha(z-z_0)}{C}=t \quad \text{とおく}$$

$$x-x_0=\frac{A}{\alpha}t \quad y-y_0=\frac{B}{\alpha}t \quad z-z_0=\frac{C}{\alpha}t$$

交点

$$Ax_0+By_0+Cz_0+\alpha t+D=0 \quad \therefore t=-\frac{1}{\alpha}(Ax_0+By_0+Cz_0+D)$$

$$\therefore (x-x_0)^2+(y-y_0)^2+(z-z_0)^2=\frac{A^2+B^2+C^2}{\alpha^4}(Ax_0+By_0+Cz_0+D)^2$$

$$\therefore \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}=\frac{1}{\alpha}|Ax_0+By_0+Cz_0+D|$$

$$=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

(ロ) 原点を通って平面 $x+2y+3z-1=0$ に垂直な直線

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}=t \qquad t+4t+9t=1 \qquad t=\frac{1}{14}$$

直線と平面の交点 $\left(\dfrac{1}{14}, \dfrac{2}{14}, \dfrac{3}{14}\right)$

$\therefore$ 求める点は $\left(\dfrac{1}{7}, \dfrac{2}{7}, \dfrac{3}{7}\right)$

2.2 $(1, 1, 2)$ $\quad \vec{A}=2\vec{i}+\vec{j}+\vec{k}$

$2(x-1)+(y-1)+(z-2)=0 \qquad 2x+y+z-5=0$

2.3 直線 $\dfrac{x-3}{-1}=\dfrac{y+1}{2}=\dfrac{z-2}{3}$，求める平面は $(3, -1, 2)$，$(3, 4, 5)$

を通り直線に平行．平面を $Ax+By+Cz+D=0$ とすると

$$\therefore \begin{cases} -A+2B+3C=0 \\ 3A-B+2C+D=0 \\ 3A+4B+5C+D=0 \end{cases}$$

$5B+3C=0$

$C=-\dfrac{5}{3}B \qquad A=2B-5B=-3B$

$\left(-9B-B-\dfrac{10}{3}B\right)+D=0$

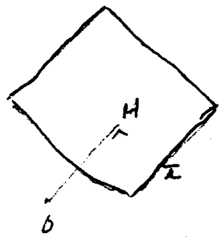
$\therefore D=\dfrac{40}{3}B$

$-3x+y-\dfrac{5}{3}z+\dfrac{40}{3}=0$

$-9x+3y-5z+40=0$

$9x-3y+5z=40$

2.4 Oより平面へ下した垂線の足をH、OH=p とすると



$\vec{OH}=(pl,\ pm,\ pn)$

π上の任意の点を $P(x, y, z)$ とすると

$OH \perp HP$ より

$\vec{OH}\cdot\vec{HP}=0$

$pl(x-pl)+pm(y-pm)+pn(z-pn)=0$

$\therefore\ lx+my+nz-p(l^2+m^2+n^2)=0$ $l^2+m^2+n^2=1$ より

$lx+my+nz=p$

2.5 $x+y+z=1$, $y-2z=2$

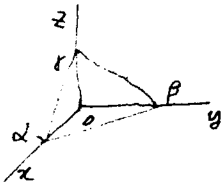
$z=\frac{y-2}{2}$ $x=1-z-y=1-(2z+2)-z$

$=-3z-1$

$\frac{x+1}{-3}=z$

$\therefore\ \frac{x+1}{-3}=\frac{y-2}{2}=z$ $(-3,\ 2,\ 1)$

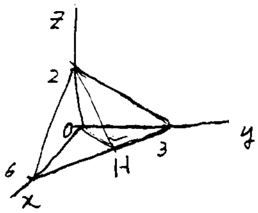
2.6



$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

$\therefore\ \frac{1}{6}\alpha\beta\gamma$

2.7



$x+2y+3z=6$

$\frac{6\times3}{2}=\frac{\sqrt{3^2+6^2}}{2}OH$ $OH=\frac{6}{\sqrt5}$

$\sqrt{2^2+\frac{36}{5}}=\sqrt{\frac{56}{5}}=2\sqrt{\frac{14}{5}}$

$\frac{1}{2}\sqrt{45}\cdot 2\sqrt{\frac{14}{5}}=3\sqrt{14}$

2.8 $l:\ x-1=\frac{y-2}{2}=\frac{z-3}{3}$ $\pi:\ x+y+z=3$

(1) $x-1=t$ とおいて

$x=t+1$, $y=2t+2$, $z=3t+3$

$6t+6=3$ $t=-\frac{1}{2}$

$P_0\left(\frac{1}{2}, 1, \frac{3}{2}\right)$

(2) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(3) l 上の点 $(1, 2, 3)$ から π への垂線の足

$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1} = t$ とおくと

$x = t+1 \quad y = t+2 \quad z = t+3$

$\therefore 3t+6 = 3 \quad t = -1 \quad \therefore x = 0,\ y = 1 \quad z = 2$

$P_0'(0, 1, 2)$

(4) $\overrightarrow{P_0P_0'} = \left(-\frac{1}{2}, 0, \frac{1}{2}\right)$

$\therefore \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

2.9 $\frac{x-a}{u} = \frac{y-b}{v} = \frac{z-c}{w} \qquad x = 0, \quad y = v\left(\frac{b}{v} - \frac{a}{u}\right) \quad z = w\left(\frac{c}{w} - \frac{a}{u}\right)$

$\left(0,\ b - \frac{av}{u},\ c - \frac{aw}{u}\right)$

2.10 $x+y+z = 1 \qquad (3, 4, 5)$

$\frac{x-3}{1} = \frac{y-4}{1} = \frac{z-5}{1} = t \qquad x = t+3 \quad y = t+4,\ z = t+5$

$3t + 12 = 1 \qquad t = -\frac{11}{3}$

$x = -\frac{2}{3} \quad y = \frac{1}{3} \quad z = \frac{4}{3} \qquad \left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

$\frac{1}{3}\sqrt{3 \cdot 11^2} = \frac{11}{3}\sqrt{3}$

2.11 $l;\ \frac{x-x_0}{1} = \frac{y-y_0}{2} = \frac{z-z_0}{3}, \quad \pi;\ x+y-z = 0 \qquad (1, 1, -1)$

$1\cdot 1 + 2\cdot 1 + 3\cdot(-1) = 0$ l は π の法線方向と垂直

$\therefore (x-x_0) + (y-y_0) - (z-z_0) = 0$

$x+y-z = x_0+y_0-z_0$

§3 球　§4 空間曲線

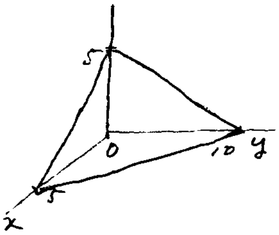
3.1　g; $\frac{x-1}{2} = \frac{y}{1} = \frac{z}{2}$　　$(1,2,3)$

(1)　α; $2(x-1) + y - 2 + 2(z-3) = 0$

$2x + y + 2z - 10 = 0$

(2)　$\frac{|2a+b+2c-10|}{3}$

(3)



$(x-a)^2 + (y-a)^2 + (z-a)^2 = a^2$

$2x + y + 2z - 10 = 0$

$\frac{|2a+a+2a-10|}{3} = a$

$5a - 10 = 3a$　　$a = 5$

$\therefore a = 5$

3.2　$x^2 + y^2 + z^2 = 14$　　$(1,2,3)$

$2(x-1) + 4(y-2) + 6(z-3) = 0$

$x + 2y + 3z = 14$

4.1　$x = a\sin^2 t$　$y = a\sin t\cos t$　$z = a\cos t$

$x' = 2a\sin t\cos t$,　$y' = a(\cos^2 t - \sin^2 t)$　$z' = -a\sin t$

法平面

$2a\sin t\cos t(x - a\sin^2 t) + a(\cos^2 t - \sin^2 t)(y - a\sin t\cos t)$

$- a\sin t(z - a\cos t) = 0$

$a\sin 2t \cdot x - a^2\sin 2t\sin^2 t + a\cos 2t\, y - a^2\frac{1}{2}\sin 2t\cos 2t$

$- z a\sin t + a^2\frac{1}{2}\sin 2t = 0$.

$x a\sin 2t + y a\cos 2t - z a\sin t - a^2(\sin 2t\sin^2 t + \frac{1}{2}\sin 2t\cos 2t - \frac{1}{2}\sin 2t)$

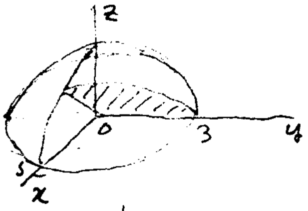
$\sin 2t\sin^2 t + \frac{1}{2}\sin 2t(\cos 2t - 1) = \sin 2t(\sin^2 t - \sin^2 t) = 0$

$\therefore ax\sin 2t + ay\cos 2t - az\sin t = 0$.

∴ 法平面は原点を通る。

5. § 5. 2次曲面

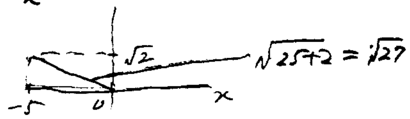
5.1 $\dfrac{x^2}{25}+\dfrac{y^2}{9}+z^2=1$ $\quad\sqrt{2}\,x+5z=0$



切り口のxy平面への正射影は

$$\frac{x^2}{25}+\frac{y^2}{9}+\frac{2x^2}{25}=1$$

$$\frac{3x^2}{25}+\frac{y^2}{9}=1$$



切り口上の点からy軸への垂線の長さをXとすると

$$x=\frac{5}{\sqrt{27}}X$$

$$\therefore\ \frac{3}{25}\,\frac{25}{27}X^2+\frac{y^2}{9}=1 \qquad \frac{X^2}{9}+\frac{y^2}{9}=1 \qquad \therefore\ X^2+y^2=9$$

原点を中心とする半径3の円

面積は 9π

5.2 (1) $Ax^2+By^2+Cz^2=1$ $\quad f(x,y,z)=Ax^2+By^2+Cz^2-1$ とおく

$f_x(x_1,y_1,z_1)=2Ax_1 \quad f_y(x_1,y_1,z_1)=2By_1 \quad f_z(x_1,y_1,z_1)=2Cz_1$

∴ 接平面は

$$Ax_1(x-x_1)+By_1(y-y_1)+Cz_1(z-z_1)=0$$

$$Ax_1x+By_1y+Cz_1z=1$$

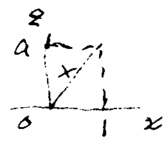
(2) 単位法線ベクトル $\dfrac{1}{\sqrt{A^2x_1^2+B^2y_1^2+C^2z_1^2}}(Ax_1,\ By_1,\ Cz_1)$

原点からの距離 $\dfrac{1}{\sqrt{A^2x_1^2+B^2y_1^2+C^2z_1^2}}$

5.3 $z^2=4x^2+y^2$ $\quad ax+z=1$

(1) T ; $(1-ax)^2=4x^2+y^2$

$$(4-a^2)x^2+2ax+y^2=1$$



(2) $(4-a^2)\left(x+\dfrac{a}{4-a^2}\right)^2+y^2=1+\dfrac{a^2}{4-a^2}$

$4-a^2=1 \quad \therefore\ a=\pm\sqrt{3}$

(3) $\dfrac{x}{X}=\dfrac{1}{\sqrt{1+a^2}} \qquad x=\dfrac{X}{\sqrt{1+a^2}}$

:6

5.3 (3) $(4-a^2)\frac{x^2}{1+a^2}+2a\frac{x}{\sqrt{1+a^2}}+y^2=1$

$\frac{4-a^2}{1+a^2}=1 \qquad 4-a^2=1+a^2 \qquad a=\pm\sqrt{\frac{3}{2}}$

5.4 点 (0, -2, 2) を通る直線

$\frac{x}{l}=\frac{y+2}{m}=\frac{z-2}{n}=t \qquad x=lt,\ y=mt-2,\ z=nt+2$

$x^2+2y^2+4z^2-8z=0$

$x^2+2y^2+4(z-1)^2=4$

$l^2t^2+2(mt-2)^2+4(nt+1)^2=4$

$(l^2+2m^2+4n^2)t^2+(-8m+8n)t+8+4-4=0$

$(l^2+2m^2+4n^2)t^2-8(m-n)t+8=0$

$\therefore\ 16(m-n)^2-8(l^2+2m^2+4n^2)=0$

$-16n^2-32mn-8l^2=0 \qquad 2n^2+4mn+l^2=0$

$2n(n+2m)+l^2=0$ 　xy 平面との交点では $z=0$

$\therefore\ t=-\frac{2}{n} \qquad \therefore\ y=\frac{-2m}{n}-2 \qquad x=-\frac{2l}{n}$

$\therefore\ m=-\frac{1}{2}n(y+2) \qquad l=-\frac{1}{2}nx$

$2n^2-2n^2(y+2)+\frac{1}{4}n^2x^2=0$

$2-2(y+2)+\frac{1}{4}x^2=0 \qquad -8-8y+x^2=0$

$y=\frac{1}{8}x^2-1$